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Multiscaling statistical procedures for the exploration of biophysical couplings in intermittent turbulence. Part I. Theory

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Abstract

Intermittency, a fundamental property of high Reynolds number turbulence, has seldom been described in ocean sciences. As a consequence, and despite several recent studies describing the intermittent distributions of temperature, salinity, nutrient concentrations, phytoplankton biomass and zooplankton abundance, the implications of intermittency on (i) the distribution of purely passive and biologically active scalars (e.g., phytoplankton cells) and (ii) biophysical couplings in the ocean are still poorly understood. We thus present both terminological and phenomenological clarification of the intermittency concept in turbulence studies. Next, univariate multifractal procedures investigating the properties of intermittent stochastic processes are presented. They characterize the statistics of intermittent variables using a set of three basic parameters in the multifractal framework, whatever the scales and the intensity. The multifractal formalism is then extended to more than one variable to investigate the degree of dependence among random fields by investigating the nature of their joint distribution. The main advantages of these unusual formalisms are that they make no assumptions about the spectrum or the distribution of data sets, fully take into account the intrinsic multiscaling properties of the data, and more generally explore qualitatively and quantitatively the correlations of large and small fluctuations of processes.

1. Introduction

Intermittency is a fundamental property of high Reynolds number turbulence, such as oceanic turbulence. Numerous experimental data analyses done in different frameworks and on different geographical regions have shown that physical and biological patterns and processes in marine sciences display high intermittency (Gibson,

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1991; Pascual et al., 1995; Seuront et al., 1996a, 1999; Jou, 1997; Jimenez, 2000). This intermittency may correspond to a basic property of aquatic ecosystem studies, as sharp, local fluctuations are ubiquitously observed in space-time distributions of turbulent dissipation rates, temperature, salinity and plankton abundance. These localized events have critical consequences at microscale (i.e. <1 m), since planctonic mating, predator-prey contacts and chemical reactions all occur at microscale. It is thus crucial to accurately describe and model intermittency and the couplings between intermittent variables (i.e. turbulent velocity and phytoplankton abundance).

In describing fully developed turbulence, classical methods often characterize the scaling properties of velocity and passive scalar fluctuations using structure functions, i.e., the statistical moments of increments of the original turbulent field, with varying time or space lags (see Monin and Yaglom, 1975; Frisch, 1995). This approach is able to characterize all the fluctuations at all scales; it can indicate the scaling regimes and scale breaks, and fully describes the probability distribution of the fluctuations of a given increment lag. Following the earlier work of Meneveau et al. (1990), we show here how such a methodology can be extended to deal with the couplings of two simultaneously recorded intermittent fields, such as velocity and temperature, or temperature and fluorescence. We characterize these couplings using a joint moment approach. In this framework couplings are studied at all scales and all intensities. This generalizes the usual correlation function, and we propose the denomination of "Generalized Correlation Function" (GCF). We theoretically provide predictions corresponding to special cases: independence, proportionality, stochastic proportionality, etc. We finally argue that such procedure should be generalized for intermittent fields, and is able to detect very particular and important couplings. GCF's effectively find and characterize the degree of biological and physical coupling. Some findings shown here may well be quite general for biophysical couplings, including identical fluctuations for medium and low intensities, and very different behaviour for high intensity fluctuations.

In this paper, we discuss the concept of intermittency, and then review the multifractal framework. We then introduce an objective technique for determining if two stochastic processes can be regarded as being independent or not, and for investigating the nature of their potential coupling.

2. Intermittency

Historically, the notion of intermittency (see Fig. 1 for examples of intermittent distributions) has presented many challenges to investigators approaching from a variability of fields. Batchelor and Townsend (1949) write that "*the basic observation which requires explanation is that activation of large wave-numbers is very evenly distributed over space*", but that "*as the wavenumber is increased the fluctuations seem to tend to an approximate on–off, or intermittent, variation*". Nearly two decades later, Stewart (1969) mentions that "*The non-Gaussian, intermittent character of the small-scale structure becomes more marked as the Reynolds number increases. It seems to be fundamental to the nature of the turbulence cascade, but as with many aspects of turbulence we do not have a fully satisfactory theoretical explanation*". In 1995, Frisch states that a process is intermittent when it "*displays activity during only a fraction of the time, which decreases with the scale under consideration*". In marine sciences, intermittency has mainly been described in terms of "uneven distributions" and "patchiness" in the plankton and has been qualitatively and quantitatively investigated over the last century (e.g., Haeckel, 1891; Hardy, 1936; Cushing, 1962; Cassie, 1963). Early scaling techniques such as autocorrelation and power spectral analyses (Platt and Denman, 1975), together with the scaling tools introduced by Mandelbrot (1977, 1983) as fractal geometry, and the multifractal cascade models of turbulence (see Frisch, 1995 for a review) that followed, have been fruitful approaches to quantifying intermittency. More recently a number of papers have developed methods for quantifying turbulent activity in the marine system as a function of both the spatial and temporal scale,

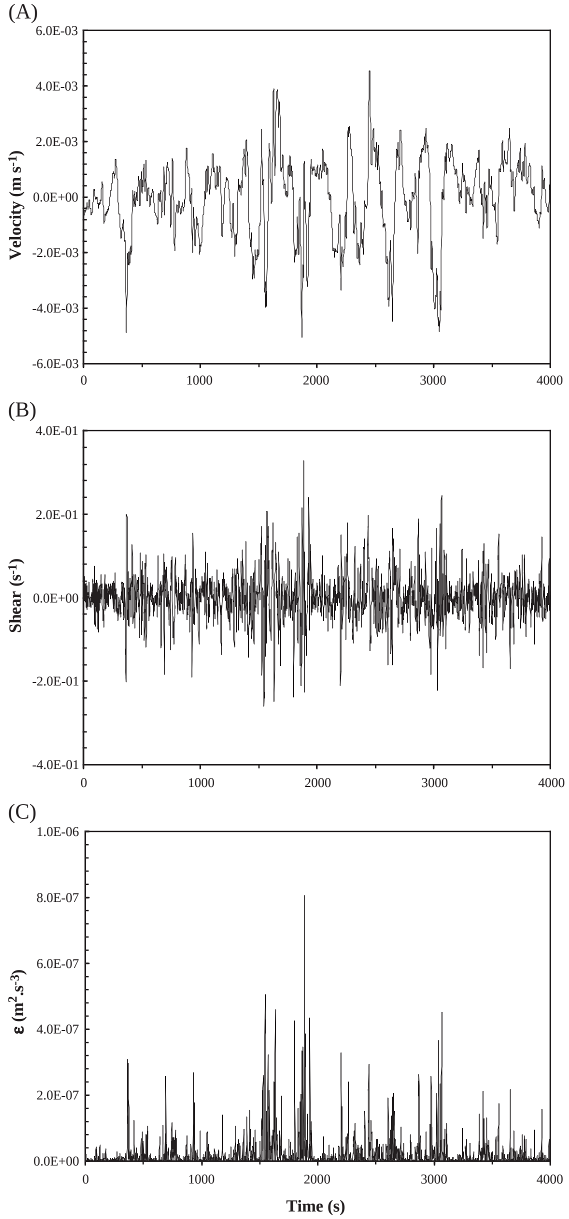

Fig. 1. Illustration of the intermittency concept using velocity (A), shear (B) and instantaneous dissipation rates (C) obtained from the ocean with a new free-fall profiler (TurboMAP, Turbulence Ocean Microstructure Acquisition Profiler) recording turbulent fluctuations at 256 Hz. Intermittency, characterized by sharp and local fluctuations, is more clearly visible in the shear and the dissipation rates signals than in the velocity signal.

the active region occupied, and their intensity; see for example Jou (1997), Seuront et al. (2001), Currie (2001) and Fisher et al. (2004). Clearly, intermittency is widely observed, but has so far escaped the confines of a narrow, precise definition.

The definition of intermittency greatly varies from author to author, leading to a non-unified framework. Jiménez (2000) states that "*intermittency is a common phenomenon in many complex systems, and is a natural consequence of cascades*". According to Svendsen (1997), the production of turbulence is not a continuous process but usually has an intermittent character and the turbulence appears as bursts. Estrada and Berdalet (1997) and Jiménez (1997), referring to the coherent nature of turbulence, define intermittency as a general feature of turbulent flows, related to the presence of strong coherent vortices, with diameters of the order of 10 times the Kolmogorov scale l_k, $l_k = (v^3/\varepsilon)^{1/4}$ where v is the kinematic viscosity ($m^2\,s^{-1}$) and ε the turbulent kinetic energy dissipation rate ($m^2\,s^{-3}$). On the other hand, the term "intermittency" as been used to characterize "*the phenomena connected with the local variability of the dissipation*" (Jiménez, 1998) as well as "*instantaneous gradients of scalars such as temperature, salinity or nutrients, greatest at scales similar to the Kolmogorov microscale*" (Gargett, 1997; Sanford, 1997). Pope (2000) defines intermittency in two distinct ways. First, describing a sharp interface between a turbulent region and a non-turbulent region, he considers that an intermittent flow is characterized by a motion that "*is sometimes turbulent and sometimes non-turbulent*". Secondly, he introduces the concept of "*internal intermittency*" to characterize the strong fluctuations perceptible in the instantaneous distribution of the dissipation rate observed by Batchelor and Townsend (1949). A more intuitive definition can be found in Seuront et al. (2001) who state that "*this form of variability reflects heterogeneous distributions with a few dense patches and a wide range of low density patches*".

For the community of engineers in fluid mechanics, intermittency is often viewed as a transition between laminar and turbulent flows.

Wilcox (1998) also mentions that "*approaching the freestream from within the boundary layer, the flow is not always turbulent. Rather, it is sometimes laminar and sometimes turbulent, i.e., it is intermittent*". The intermittency phenomenon is also often mixed up with its statistical consequences. For instance, Seuront et al. (2001) consider that in an intermittent framework "*we occasionally should expect stronger bursts which accentuate the skewness of a given probability distribution*". Similarly, Frisch (1995) considers that "*the kurtosis is a useful measure of intermittency for signals having a bursty aspect*".

If the common factor emerging from the examples described above seems to be the irregular and unpredictable nature of intermittent distributions, we stress the need to clarify what is meant, or at least what we mean here, by "intermittency". Pope (2000) argues that intermittency is a dissipation-range phenomenon, leading to very violent events that cannot be smoothed out until the external scale is reached. These dissipation-range intermittent bursts are perceptible in the whole inertial subrange. Thus, intermittency is found at all scales, as other turbulent phenomena. Intermittency can also be regarded, as suggested by Jou (1997), as strong fluctuations of the energy transfer between eddies of different scales. We will see in the next section that this assertion is fully congruent with the phenomenological way to build turbulent cascade models. Let us also note that a potential reason for intermittency is the presence of strong coherent vortices, with diameters of the order of 10 times the Kolmogorov scale (Siggia, 1991; Jiménez et al., 1993; Jiménez and Wray, 1994). An extensive discussion of the coherent nature of turbulent flows can be found in Frisch (1995).

While most of the above cited references specifically referred to intermittency in the framework of turbulent flows, we stress that a general consensus can be reached if one considers that a given pattern/process is intermittent in structure if (i) it is characterized by sharp fluctuations, (ii) it is responsible for a skewed probability distribution, and (iii) it has a long-term memory signature, perceptible from the power law form of its autocorrelation function.

3. Characterizing univariate intermittent distribution: a review of the multifractal framework

3.1. Scaling relations for velocity and passive scalars

Scaling relations for turbulent velocity and passive scalar (originally temperature) fields have been expressed in Eulerian turbulence using the energy flux ε as (Kolmogorov, 1941; Obukhov, 1941):

$$\varepsilon_l \approx \frac{(\Delta V_l)^3}{l} \tag{1}$$

and the scalar variance flux χ as (Obukhov, 1941, 1949; Corrsin, 1951):

$$\chi_l \approx \frac{(\Delta S_l)^2 \Delta V_l}{l}, \tag{2}$$

where $\Delta V_l = |V(x+l) - V(x)|$ and $\Delta S_l = |S(x+l) - S(x)|$ are the velocity shear and passive scalar gradient at scale l, and $\Delta V_l/l$ is the inverse of the local eddy turnover time. Originally, these scaling relations were considered in the framework of homogeneous turbulence, i.e. the fluxes ε_l and χ_l were considered as homogeneous, exhibiting no scale dependence. As a consequence, a unique exponent was required for the velocity and passive scalar, the so-called $\frac{1}{3}$ law in physical space:

$$\Delta V_l \approx l^{1/3}, \tag{3}$$

$$\Delta S_l \approx l^{1/3}. \tag{4}$$

In Fourier space, widely used in ecology to separate and measure the amount of variability occurring at different frequencies and wavenumbers, assuming local isotropy and three-dimensional homogeneity of turbulence in the inertial subrange, Eqs. (3) and (4) can be rewritten to describe the velocity fluctuations and the fluctuations of a passive scalar using the spectral densities $E_V(k)$ and $E_S(k)$ as:

$$E_V(k) \approx k^{-\beta_V}, \tag{5}$$

$$E_S(k) \approx k^{-\beta_S}, \tag{6}$$

where k is equally a frequency or a wavenumber whether velocity and passive scalar fluctuations are considered in time or in space, and β_V and β_S are characteristic spectral exponents defined as $\beta_V = \beta_S = 5/3$ (Fig. 2).

To move beyond the somewhat restrictive cases in which the assumption of homogeneity strictly applies, requires taking into account the intermittency of the system. Intermittency is ubiquitous in the ocean at the most commonly observed scales. The subscript l in Eqs. (1)–(4) accounts for the scale dependence of the intermittent turbulent fluxes. Since many of the assumptions underlying traditional spectral analysis can be restrictive, given the typical conditions encountered in marine systems, methods that generalize the approach of modelling random variability (e.g., Yamazaki and Okubo, 1995; Visser, 1997) are investigated here. In the multifractal framework, such methods allow the intermittency of turbulent velocity and passive scalar fluctuations to be fully taken into account (Seuront et al., 1999, 2002, 2005).

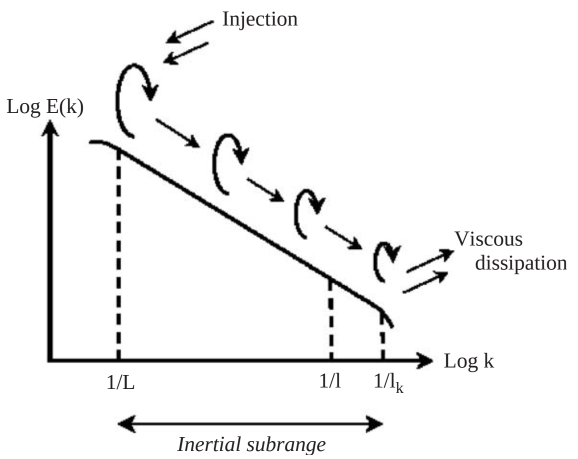

Fig. 2. Schematic representation showing the form of the energy spectrum of turbulent velocity cascade, where $E(k)$ is the spectral density and k is a wavenumber (m^{-1}). The kinetic energy generated at large scale L cascades through the inertial subrange through a hierarchy of eddies of decreasing size down to the viscous Kolmogorov scale l_k, where it is dissipated into heat. Practically, this cascade is observed between the outer scale L, and the resolution scale l of the measurements (often limited by the size and/or the sampling frequency of the sampling apparatus), leading to the scale ratio $\lambda = L/l$. The slope of the linear behavior of the spectral density versus the wavenumber in a log–log plot provides an estimate of the so-called spectral exponent β. One must also note here that because time and space are linked through the 'Taylor's hypothesis of frozen turbulence' which states that the temporal and spatial scales t and l are related by a constant velocity v ($v = l/t$) the spectral density can equivalently be expressed as a function of the frequency f (s^{-1}).

In cascade models of turbulence (e.g., Yaglom, 1966; Frisch et al., 1978; Benzi et al., 1984; Schertzer and Lovejoy, 1983, 1987; Meneveau and Sreenivasan, 1987; Yamazaki, 1990; Saito, 1992; She and Levêque, 1994; see also Frisch, 1995, and Seuront et al., 2005 for reviews), the highly intermittent fluxes are the results of a multiplicative process in which the variability is built up from large to small scales: larger structures are multiplicatively randomly modulated by smaller scales. This leads to multifractal fields, with the following multiscaling statistics (Schertzer and Lovejoy, 1987; Schmitt et al., 1996):

$$\langle(\varepsilon_l)^q\rangle \approx \lambda^{K_\varepsilon(q)} \approx l^{-K_\varepsilon(q)}, \tag{7}$$

$$\langle(\chi_l)^q\rangle \approx \lambda^{K_\chi(q)} \approx l^{-K_\chi(q)}, \tag{8}$$

$$\langle[\Delta V_l]^q\rangle \approx \lambda^{-\zeta_V(q)} \approx l^{\zeta_V(q)}, \tag{9}$$

$$\langle[(\Delta S_l)^2 \Delta V_l]^q\rangle \approx \lambda^{-\zeta_{V,S}(3q)} \approx l^{\zeta_{V,S}(3q)}, \tag{10}$$

where the angle brackets "$\langle \cdot \rangle$" indicate ensemble (statistical) averaging, λ the scale ratio between the largest external scale L and the actual scale l (i.e. $\lambda = L/l$), $K_\varepsilon(q)$ and $K_\chi(q)$ the scaling moment functions for the fluxes ε_l and χ_l, $\zeta_V(q)$ and $\zeta_{V,S}(q)$ the scaling moment functions of the velocity structure function and the joint structure function scaling exponent of the product $(\Delta S_l)^2 \Delta V_l$. Using Eqs. (1) and (2), the functions $K_\varepsilon(q)$ and $K_\chi(q)$ are defined as:

$$K_\varepsilon(q) = q - \zeta_V(3q), \tag{11}$$

$$K_\chi(q) = q - \zeta_{V,S}(3q). \tag{12}$$

Because the fluxes are conserved by the equation of motion over the inertial subrange, they are assumed in this framework to be independent of scale (i.e., strictly scale invariant):

$$\langle\varepsilon_l\rangle = \langle\varepsilon_1\rangle, \tag{13}$$

$$\langle\chi_l\rangle = \langle\chi_1\rangle. \tag{14}$$

It subsequently appears from Eqs. (7) and (8) that

$$K_\varepsilon(1) = 0, \tag{15}$$

$$K_\chi(1) = 0. \quad (16)$$

Such multifractal fields are called ‘‘conservative multifractals’’. The conservation of the fluxes ε_l and χ_l implies

$$\zeta_V(3) = 1, \quad (17)$$

$$\zeta_{V,S}(3) = 1, \quad (18)$$

which corresponds to the exact relations for the small-scale dissipation fields given by Kolmogorov (1941) and Yaglom (1949). The scaling moment functions $K_\varepsilon(q)$, $K_\chi(q)$, $\zeta_V(q)$ and $\zeta_{V,S}(q)$ characterize all the fluctuations of the fluxes of energy and scalar variance, and the fluctuations of the velocity shear and scalar gradient. In other words, as under fairly general conditions the probability distribution of a random variable is equivalently specified by its statistical moments, the scaling moment functions $K(q)$ and $\zeta(q)$ describe the scale dependence of the statistical distributions.

However, the functions $K(q)$ and $\zeta(q)$ do not provide the scaling moment function $\zeta_S(q)$ of the passive scalar fluctuations defined as

$$\langle[\Delta S_l]^q\rangle \approx \lambda^{-\zeta_S(q)}. \quad (19)$$

The corresponding flux $\varphi_l = \varepsilon_l^{-1/2}\chi_l^{3/2}$ is indeed a mixed flux of energy and scalar variance, which is non-conservative. The two fluxes ε_l and χ_l are intrinsically correlated, so they cannot be assumed independent (Benzi et al., 1992). Alternatively, the mixed flux may be related to the structure function of velocity fluctuations and scalar gradients as (Schmitt et al., 1996):

$$\zeta_V(q) = q/3 + K_\varepsilon(q/6) - K_\chi(q/2) \quad (20)$$

and

$$\zeta_S(q) = \zeta_{V,S}(3q/2) - \zeta_V(q/2). \quad (21)$$

For monoscaling (i.e. monofractal) processes, the function $\zeta(q)$ is linear: $\zeta(q) = q/2$ for Brownian motion, and $\zeta(q) = q/3$ for homogeneous turbulence. For multiscaling processes, this exponent is non-linear and concave. Figs. 3 and 4 provide a step-by-step analysis from spectral analysis to structure function analysis for turbulent velocity and in vivo fluorescence fluctuations, to illustrate the above described concepts and to demonstrate the generality of their applicability.

3.2. *Cascade models for turbulent fluxes*

Since the first lognormal proposal of Kolmogorov (1962) and Obukhov (1962), and the first explicit cascade model of Yaglom (1966), many different cascade models (e.g., Fig. 2) have been proposed to represent intermittent fluxes; see Seuront et al. (2005) for a review of cascade models in turbulence. We review here quickly many of these models. A first family of models is composed of discrete models, for which the scale ratio between a structure and the daughter structure is a discrete integer. Due to their discrete nature, these models are not realistic, but have been introduced for their simplicity and ability to reproduce experimental intermittency. These models include the mono-fractal β-model (Novikov and Stewart, 1964; Mandelbrot, 1974; Frisch et al., 1978), the α-model (Schertzer and Lovejoy, 1983), the p-model (Meneveau and Sreenivasan, 1987) and the random β-model (Benzi et al., 1984). A detailed review of these models may be found in Paladin and Vulpiani (1987), Meneveau and Sreenivasan (1991), Frisch (1995), Schmitt (2001) and Seuront et al. (2005).

A more realistic family of models is composed of ‘‘continuous cascades’’ corresponding to log-infinitely divisible (log-ID) stochastic models. The idea of a continuous scale dilatation for the cascade process has already been recognized by Novikov (1969). He explicitly showed later (Novikov, 1990, 1994) that this corresponds to choosing for the logarithm of the cascade process, infinitely divisible random processes. Infinite divisibility is a property of probability laws (see e.g., Feller, 1971) characterized by the fact that any random variable belonging to this law may be written as a sum of an arbitrarily large number of independent random variables having each the same law (independent identically distributed). This property considerably restricts available probability laws: the choice of continuous models possessing the log-ID property is still large but much less than for discrete models. The most well-known ID laws are the Gaussian, Lévy-stable, Poisson and Gamma. We review here the corresponding log-ID continuous cascade models that have been advocated in various publications in turbulence.

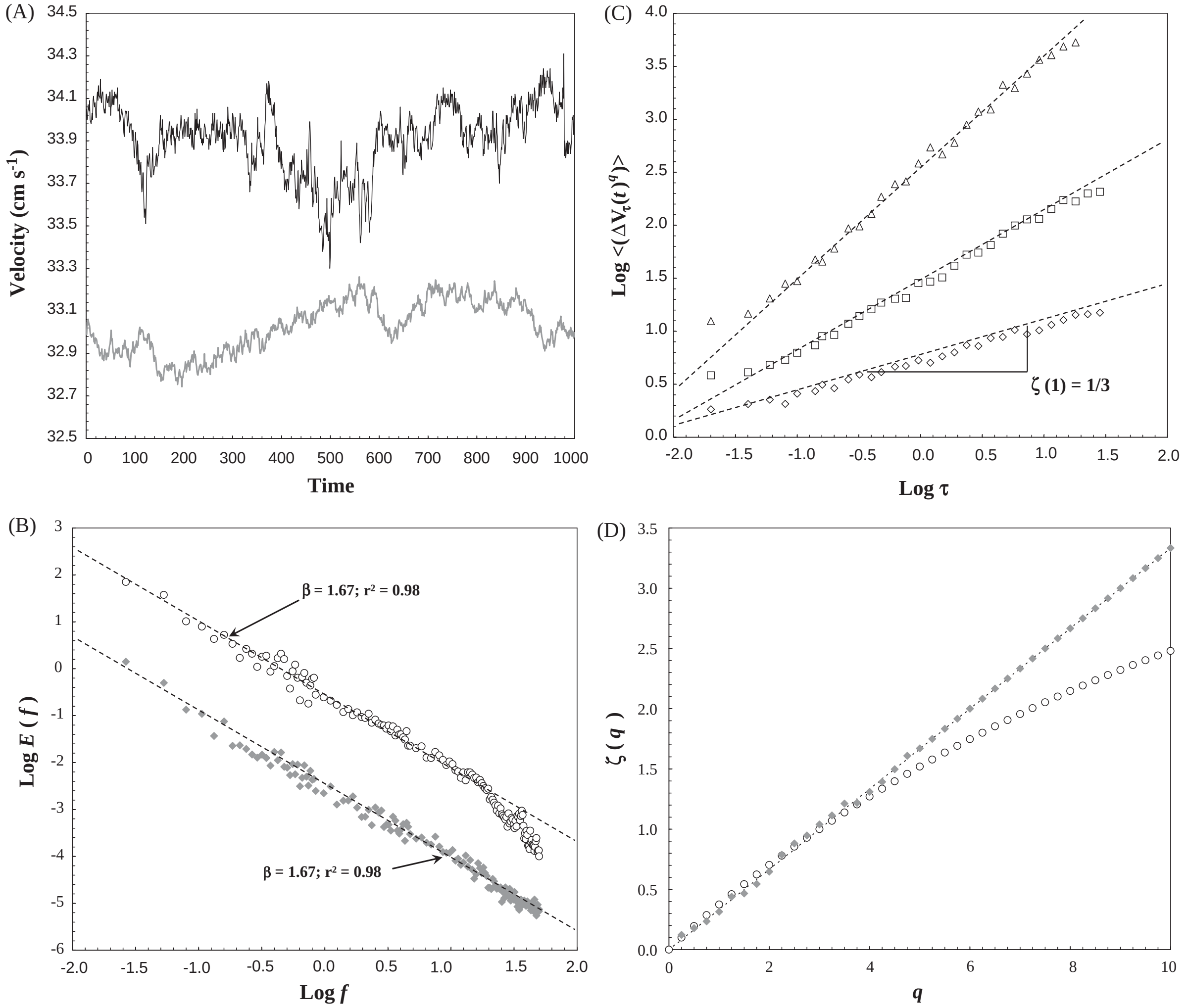

Fig. 3. Step-by-step analysis of a time series of grid-generated turbulent velocity recorded by hot-wire velocimetry at 100 Hz in a circular flume (Seuront et al., 2004; A, black) and a synthetic time series with the same spectral properties than the empirical one (A, grey). While the empirical time series is clearly more intermittent than the synthetic one, their power spectra are fairly similar, showing a clear scaling behavior over the whole range of available scales, except at the smallest scales for the empirical one because of the electronic limitations of the instrument (B). This is confirmed by structure function analysis (C). The slopes of the log–log plots of $\langle(\Delta V(t)_\tau)^q\rangle$ versus τ shown for the empirical time series for different values of q (here $q = 1$, 2 and 3 from bottom to top) provide estimates of the functions $\zeta(q)$. The non-linear (i.e. intermittent) behavior of the functions $\zeta(q)$ estimated for the empirical time series contrasts with the linearity of the functions estimated from synthetic data that cannot be distinguished from the theoretical, non-intermittent case where $\zeta(q) = q/3$.

The lognormal model was the first scaling proposal for intermittent turbulence (Kolmogorov, 1962; Obukhov, 1962). This corresponds to a quadratic form for $\zeta_V(q)$; the condition given by Eq. (17) together with the condition $\zeta_V(0) = 0$ provides an expression for $\zeta_V(q)$ depending on only 1 parameter, the intermittency parameter $\mu = K_\varepsilon(2)$:

$$\zeta_V(q) = \frac{q}{3} - \frac{\mu}{2}\left(\left(\frac{q}{3}\right)^2 - \frac{q}{3}\right). \tag{22}$$

The lognormal model for the velocity field or for the dissipation in turbulence has been advocated in several papers (see e.g., Arneodo et al., 1998;

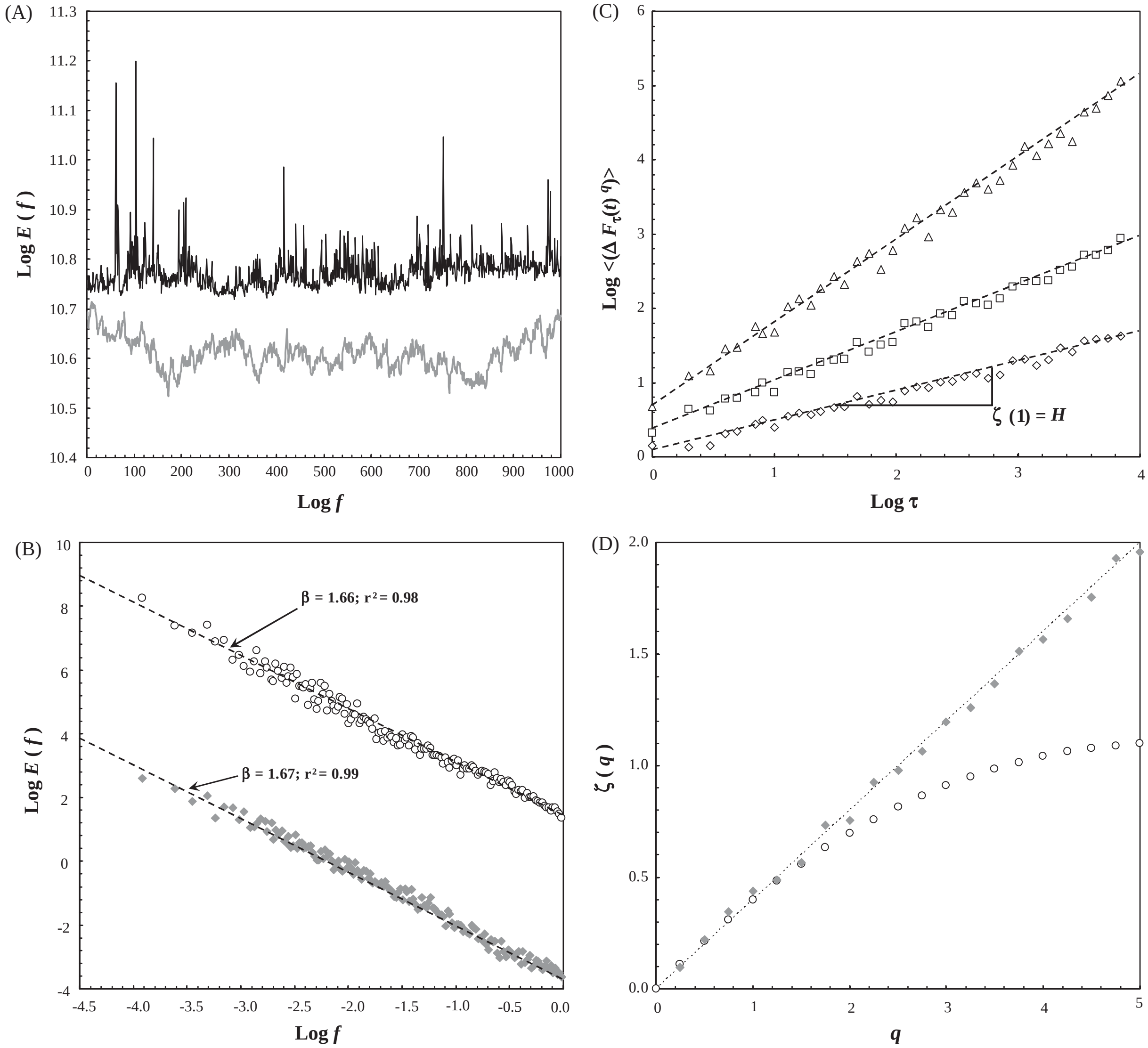

Fig. 4. Step-by-step analysis of a time series of in vivo fluorescence recorded in situ at 2 Hz in the offshore waters of the Eastern English Channel (Seuront, 1999; A, black) and a synthetic time series with the same spectral properties than the empirical one (A, grey). While the empirical time series is clearly more intermittent than the synthetic one, their power spectra are fairly similar, showing a clear scaling behavior over the whole range of available scales (B). This is confirmed by structure function analysis (C). The slopes of the log–log plots of $\langle(\Delta F_\tau)^q\rangle$ versus τ shown for the empirical time series for different values of q (here $q = 1$, 2 and 3 from bottom to top) provide estimates of the functions $\zeta(q)$. The non-linear (i.e.) intermittent behavior of the functions $\zeta(q)$ estimated for in situ time series contrasts with the linear linearity of the functions estimated from synthetic data that cannot be distinguished from the theoretical, non-intermittent case where $\zeta(q) = qH$.

Malecot et al., 2000; Chanal et al., 2000; Delour et al., 2001).

The Gaussian law belongs in fact to the Lévy-stable family corresponding to stable and attractive processes under addition (see Feller, 1971). Correspondingly, the lognormal cascade may be generalized to log-stable cascades, as proposed originally by Schertzer and Lovejoy (1987) and later by Kida (1991). The expression for the scaling exponent is of the form $\zeta(q) = Aq - Bq^{\alpha}$. For turbulent velocity, the normalization conditions $\zeta_V(3) = 1$, provides an explicit expression, as a generalization of the lognormal process:

$$\zeta_V(q) = \frac{q}{3} - \frac{C_{1\varepsilon}}{\alpha_\varepsilon - 1}\left(\left(\frac{q}{3}\right)^{\alpha_\varepsilon} - \frac{q}{3}\right), \tag{23}$$

where α ($0 \leqslant \alpha \leqslant 2$) is the basic parameter of this family; $\alpha = 2$ recovers the lognormal model. The other parameter $C_{1\varepsilon}$ is also an intermittency parameter and characterizes the fractal dimension of the mean.

Another popular model is the log-Poisson model. It was first proposed by She and Leveque (1994) and soon recognized as a log-Poisson model (Dubrulle, 1994; She and Waymire, 1995). It was also advocated in Castaing and Dubrulle (1995) and Dubrulle (1996), among others. For this model, the equation is the following:

$$\zeta_V(q) = \frac{q}{3}\,(1 - c(1-\beta)) + c(1 - \beta^{q/3}). \tag{24}$$

It depends on two parameters $0 \leqslant \beta \leqslant 1$ and $0 \leqslant c \leqslant 1$ that can be estimated experimentally or proposed based on specific hypothesis.

We may note that no general consensus has yet emerged concerning the best model for intermittent fluctuations in velocity or passive scalar turbulence. Each of these models is advocated by different group of authors, but a final answer will need new theories or new data.

3.3. General properties

As stated above, the function $\zeta_V(q)$ is non-linear and concave (Figs. 3D and 4D), while the function $K_\varepsilon(q)$ is non-linear and convex (Fig. 5A). Note from Eqs. (22) and (23) that the function $K_\varepsilon(q)$ can be thought as the intermittency correction to the non-intermittent case, $\zeta_V(q) = q/3$. For a passive scalar S, the scale invariant moment function may be written as

$$\zeta_S(q) = qH - K_\varphi(q), \tag{25}$$

where $\zeta_S(q)$ is estimated as the slope of the best linear fit of $\langle[\Delta S_l]^q\rangle$ versus the scale λ in a log–log plot; see Eq. (19) and Fig. 3. $K_\varphi(q)$ characterizes the mixed flux $\varphi_l = \varepsilon_l^{-1/2}\chi_l^{3/2}$. The functions $\zeta_S(q)$ and $K_\varphi(q)$ are non-linear and concave (Fig. 3D), and non-linear and convex (Fig. 5B), respectively. Note from Eq. (25) that the function $K_\varphi(q)$ can be thought as the intermittency correction to the non-intermittent case, $\zeta_S(q) = qH$. H is the degree of non-conservation of the average process: $H = 0$ for a conservative process (i.e. scale-independent) and $H \neq 0$ for a non-conservative process (i.e. scale-dependent). H is given by $H = \zeta_S(1)$, while it can be seen from Eq. (25) that $\zeta_V(1) > 1/3$.

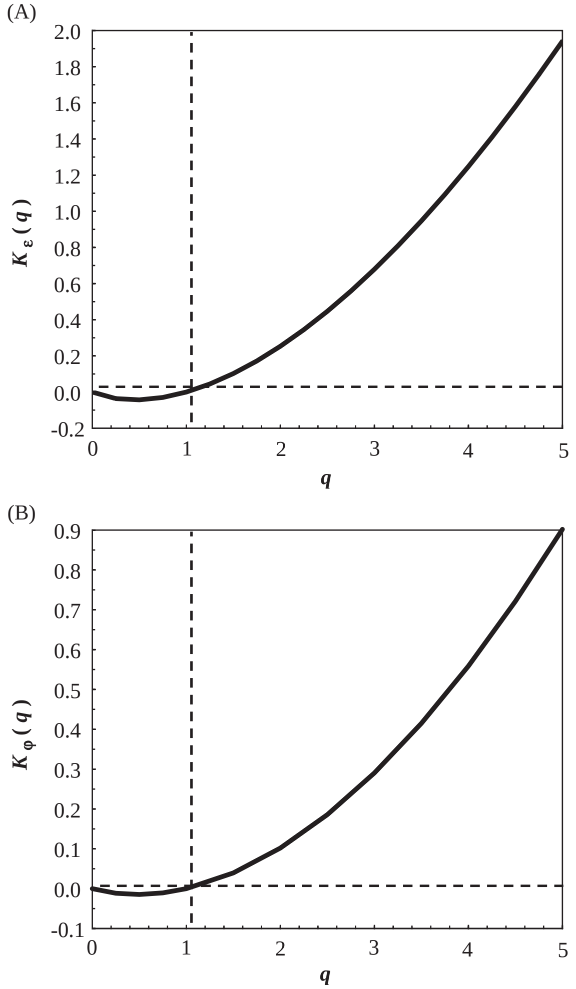

Fig. 5. Illustration of the shape of the function $K_\varepsilon(q)$ (A) and $K_\varphi(q)$ (B) estimated from the turbulent velocity and in vivo fluorescence time series presented in Figs. 3 and 4, respectively. Note that in both case the conditions $K_\varepsilon(1) = 0$ and $K_\varphi(1) = 0$ are respected.

Finally, we note here that in the multiscaling framework, intermittency is taken into account noting that:

$$\beta_V = 1 + \zeta_V(2), \tag{26}$$

$$\beta_S = 1 + \zeta_S(2). \tag{27}$$

We thus see that the intermittency corrections introduced by respectively the second term of Eqs. (23) and (25) leads to $\beta_V > 5/3$; see Seuront et al. (2005) for further details.

4. Characterizing bivariate intermittent distributions: the "Generalized Correlation Functions"

4.1. Definition

Standard procedures testing for independence between two given processes are generally based on second-order statistics (i.e. covariance and correlation functions), even when they are conducted in a scaling framework related to spectral analysis (Legendre and Legendre, 1998) or geostatistical analysis (Kitanidis, 1997). More recent procedures are based on probability density functions examination (Lueck and Wolk, 1999). The former are often implicitly based on Gaussian framework, for which non-correlation implies independence. The latter do not deal with the intrinsic multiscaling properties of intermittent fields.

We then propose here a new testing procedure based on joint moments; this can be seen as a high-order generalization of the usual correlation between two variables X and Y. It is based on some ideas given in Meneveau et al. (1990). In this paper, joint multifractal measures were studied both theoretically and experimentally. They considered mainly mixed moments in the framework of two cascade models: lognormal cascades and binomial cascades; they estimated the fractal dimension of mixed singularities, instead of scale-invariant moment functions as we choose here. Furthermore, they did not normalize joint moments as we do below, to provide joint correlations. The joint correlation functions using structure functions that we propose here are then a continuation and a development of this early study. Joint moments for scaling structure functions have been later proposed in the field of econophysics, with a generalization of the lognormal multifractal framework to multivariate lognormal multifractals. The idea was to study correlations for multiple assets, in order to characterize their return distributions; see e.g., Muzy et al. (2001). However, the final objective of such a study is portfolio optimisation, which is different from our analysis of the generalized correlation between two multifractal fields.

We now turn to the presentation of our statistical procedure to characterize the relations between two intermittent processes. Instead of random variables X and Y, we consider here the increments of two stochastic processes ΔX_l and ΔY_l (e.g., Parzen, 1962). For more convenience, let us note $x = \Delta X_l = \mathrm{e}^{G_1}$ and $y = \Delta Y_l = \mathrm{e}^{G_2}$. The joint moments can be written as the moments of a vectorial process:

$$\langle x^p y^q \rangle = \langle \mathrm{e}^{pG_1 + qG_2} \rangle = \langle \mathrm{e}^{\vec{Q}.\vec{G}} \rangle \propto l^{S(\vec{Q})}, \tag{28}$$

where the moment vector $\vec{Q}$ and the singularity vector $\vec{G}$ are, respectively, given by $\vec{Q} = (p, q)$ and $\vec{G} = (G_1, G_2)$, and the exponents $S(\vec{Q})$ characterize the multiscaling properties of the joint moments $\langle x^p y^q \rangle$. This was originally proposed in Meneveau et al. (1990), without the normalization we introduce here, defining a "Generalized Correlation Function" (GCF hereafter). The normalization of the joint moments is given as

$$c(p,q) = \frac{\langle x^p y^q \rangle}{\langle x^p \rangle \langle y^q \rangle} \propto l^{-r(p,q)}. \tag{29}$$

The "Generalized Correlation Exponent" (GCE hereafter), estimated as the slope of the linear trend of $c(p,q)$ vs. l in a log–log plot, is then expressed as

$$r(p,q) = \zeta_X(p) + \zeta_Y(q) - S(p,q), \tag{30}$$

where $\zeta_X(p)$ and $\zeta_Y(q)$ characterize the multiscaling properties of the single fluctuations $\langle x^p \rangle$ and $\langle y^q \rangle$ as defined in Eqs. (9) and (19), and $S(p,q)$ characterize the multiscaling properties of the joint fluctuations $\langle x^p y^q \rangle$; see Eq. (28). Both $c(p,q)$ and $r(p,q)$ are generalizations of correlation functions.

They express the correlation between x^p and y^q, and their scale and moment dependence. In the particular case $p=q=1$, Eq. (28) recovers the classical expression of the correlation coefficient between x and y. We nevertheless need to recall here that, whereas independence implies non-correlation, non-correlation does not imply independence.

Indeed, non-correlation corresponds simply to the relation $r(1,1)=0$. Non-correlation implies independence only in special cases such as for Gaussian processes. To show this, let us consider the joint scaling function for lognormal multifractals x and y. Using results for multivariate Gaussian processes (see any text book on multivariate stochastic processes; e.g., Samorodnitsky and Taqqu, 1994), one has the general expression for a lognormal process (see also Meveveau et al., 1990):

$$S(p,q) = a_1 p + a_2 q - a_3 q^2 - a_4 p^2 - \sigma pq \quad (31)$$

so that

$$r(p,q) = S(p,0) + S(0,q) - S(p,q) \quad (32)$$

giving

$$r(p,q) = \sigma pq. \quad (33)$$

In this case, it is clear that r(1,1) or r(2,2) is enough to estimate the only needed parameter, namely the correlation coefficient σ, so that if $r(1,1)=0$ or $r(2,2)=0$ it can be concluded that the two processes are independent ($\sigma=0$ and $r=0$ for all p and q). In the general case, this is no longer true: independence between the stochastic processes x and y means that the GCE verifies $r(p,q)=0$ whatever the values of p and q, while uncorrelation corresponds to $r(1,1)=0$. Fig. 6 thus shows the GCF, $c(p,q)$, plotted in log–log plot versus the time scale τ, for the grid-generated turbulent velocity time series and the in vivo fluorescence time series showed in Figs. 3 and 4, respectively. As these time series have been independently sampled, respectively in the laboratory and in the field, they represent an archetypical example of two independent multifractal processes. The very low values taken by the functions $c(p,q)$ indicate the absence of any correlation between the turbulent velocity and fluorescence fluctuations, ΔV_τ and ΔF_{τ}. This is confirmed and specified by the related values of the function $r(p,q)$, which remain close to zero, whatever the combinations of p and q values (Fig. 7); see Seuront and Schmitt (2005) for illustrations on the different values taken by the function $r(p,q)$ in the presence and absence of correlation between two multifractal fields.

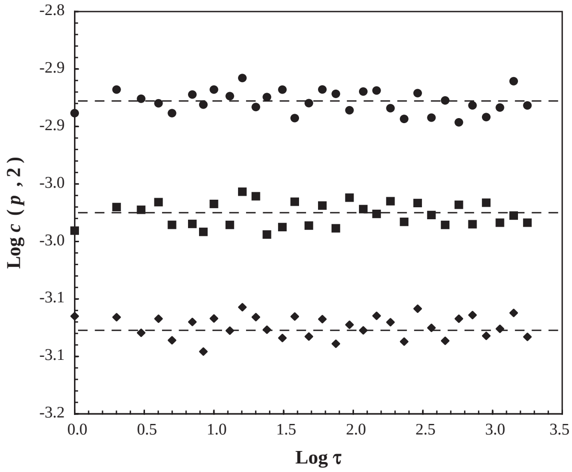

Fig. 6. The Generalized Correlation Functions (GCF) $c(p,q)$ versus the time scale τ in log–log plots, for the grid-generated turbulent velocity time series and the in vivo fluorescence time series showed in Figs. 3 and 4, respectively. The function $c(p,q)$ shown here have been estimated for a constant value of the statistical order of moment q of velocity fluctuations ($q=2$), and various values of the statistical order of moment p of in vivo fluorescence (i.e., $p=1$, 2 and 3, from bottom to top). The slopes of the linear regression estimated over the scaling ranges (dashed lines) provide estimates of the Generalized Correlation Exponents (GCE) $r(p,q)$.

4.2. Generalized correlation exponents in special cases

The function $c(p,q)$ and its related scaling exponent $r(p,q)$ can be used as an analysis tool to study the couplings between two multifractal fields x and y. To provide some basis for discussion and interpretation of experimental results, let us consider below first some limit cases, before discussing some intensity-dependent couplings and the resulting expression of the generalized correlation exponent.

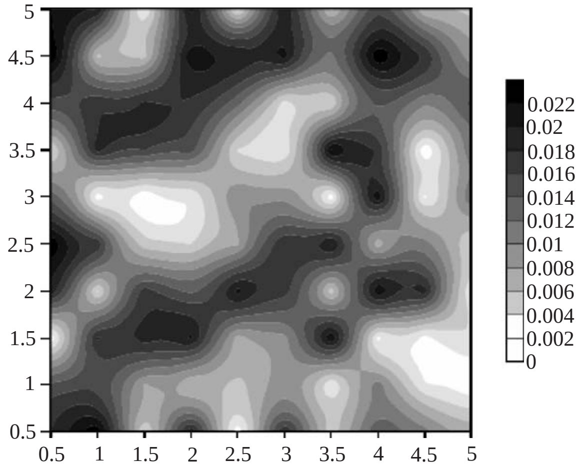

Fig. 7. The Generalized Correlation Exponents (GCE) $r(p,q)$, shown as a function of both p and q, which characterize turbulent velocity and in vivo fluorescence fluctuations, respectively. The function $r(p,q)$ is estimated here between independently sampled time series of turbulent velocity and fluorescence, respectively in the laboratory and in the field.

If x and y are independent, as what was said above, $r(p,q)=0$. On the other hand, in case of perfect proportionality $x=Ky$, where K is a constant, or for "random proportionality" $x=\kappa y$, where κ is a random variable independent on y, it is readily seen that

$$r(p,q)=\zeta_Y(p)+\zeta_Y(q)-\zeta_Y(p+q). \tag{34}$$

In particular, one may note that $r(p,q)>0$ due to the convexity of the scaling functions $\zeta(p)$. This relation can be directly tested to verify the proportionality hypothesis. Furthermore, the shape of the surface obtained is symmetric in the p–q plane. In this specific case, the function $r(p,q)$ has thus the desirable advantage to reduce considerably the number of data points, i.e. $r(p,q)$ values, needed to understand the relationship between the fields x and y.

Other simple situations may be considered: if $x=Ky^b$ with $b>0$ and K constant, or if $x=\kappa y^b$ with κ random and independent of y, then one has

$$r(p,q)=\zeta_Y(bp)+\zeta_Y(q)-\zeta_Y(bp+q). \tag{35}$$

$r(p,q)$ in Eq. (35) is still positive, but no more symmetric in the p–q plane; it is symmetric in the bp–q plane. In this framework, the value of b may be first estimated as the positive value such that

$$S(p,0)=S(0,bp). \tag{36}$$

Using the value estimated this way, this framework is then tested by verifying that $r(p/b,q)$ is indeed symmetric in the p–q plane. More generally speaking, the more $r(p,q)$ is positive, the more the $x=\Delta X_l$ and $y=\Delta Y_l$ are dependent random variables.

The main advantages of this framework are the following: it makes no assumptions about the spectrum or the distribution of data sets; it takes fully into account their intrinsic multiscaling properties in the inertial subrange scales, as well as in any range of scales characterized by a multiscaling behavior; and it may be used to characterize the nature of couplings between two fields.

5. Discussion and conclusions

Historically, intermittency has seldom been described as such in marine sciences. In physical oceanography, intermittency has mainly been discussed in terms of its consequences on sampling, data processing and statistics (Baker and Gibson, 1987; Bohle-Carbonel, 1992; Yamazaki, 1990), and is even not referred to in specialized monographs (e.g., Pond and Pickard, 1983; Mann and Lazier, 1991; Summerhayes and Thorpe, 1996; Kantha and Clayson, 2000). The situation is similar in marine ecology where turbulent intermittency and its potential effects often are not discussed. Let us mention however that turbulence intermittency has been considered as irrelevant to marine life. We may cite Estrada and Berdalet (1997): intermittent events "*should be very intense from the point of view of plankton, but calculations show that their probability is small*". Jiménez (1997) is more precise, considering that intermittent bursts "*must certainly be spectacular events from the point of view of plankton, comparable to the passing of a tornado at our scale, and probably with similar consequences on the individual involved*" but that "*they are sufficiently rare that they can be neglected in most calculations*".

In biological oceanography, patchiness, variability, heterogeneity and intermittency (see Seuront and Lagadeuc, 2001a for terminological details) has been a major issue since the early studies (e.g., Haeckel, 1891; Hardy, 1936; Cassie, 1959a, b; Cushing, 1962; Cassie, 1963; Wiebe, 1970; Fasham, 1978). Since then, instruments designed to run underway and measure changes in plankton abundance and composition have become common and are continuously improved, e.g., fluorometers (Wesson et al., 1999; Franks and Jaffe, 2001; Leboulanger et al., 2002), video plankton recorders (Lenz et al., 1995; Tiselius, 1998), high-frequency acoustics (Coyle et al., 1998; Warren et al., 2002; Wiebe et al., 2002), continuous plankton recorders (Batten et al., 2003; Johns et al., 2003), and small-scale sampling systems (Waters and Mitchell, 2002; Waters et al., 2003). Uneven plankton distributions have subsequently been widely described (e.g., Mackas and Boyd, 1979; Denman and Powell, 1984; Bennett and Denman, 1985; Mackas et al., 1985; Davis et al., 1991; Daly and Smith, 1993; Currie et al., 1998), but intermittency, as defined in the present work, has seldom been quantified. There is now more and more experimental evidence of the intermittent and multifractal nature of plankton distributions (Pascual et al., 1995; Seuront et al., 1996a, b, 1999, 2002; Seuront and Lagadeuc, 2001b; Currie, 2001; Lovejoy et al., 2001; Seuront and Schmitt, 2004). Potential practical applications to marine ecosystems have nevertheless been ignored until recently (Seuront, 2001; Seuront et al., 2001; Yamazaki et al., 2001). Theoretical considerations based upon detailed (multifractal) descriptions of the intermittency of turbulent kinetic energy dissipation rates and phytoplankton cells distributions nevertheless suggests that taking into account intermittency for critical processes such as predator–prey encounter rates, nutrient fluxes around phytoplankton cells, phytoplankton coagulation and the related size of phytoplankton aggregates and vertical fluxes has consequences far from being negligible (Seuront, 2001; Seuront et al., 2001).

We expect the theoretical framework presented here to provide additional valuable insights into our understanding of intermittency and its integration into ecologically relevant processes. As the Generalized Correlation Functions and Exponents provide an objective way to investigate the potential couplings existing between two simultaneously sampled intermittent parameters, it would help to reveal experimentally some of the phenomenology behind observed plankton distributions. In particular, previous qualitative results showing that large phytoplankton fluctuations are linked, under strong enough turbulent conditions, to weak temperature gradients and vice versa (Desiderio et al., 1993; Wolk et al., 2002), could be confirmed and quantified in the GCF/GCE framework. As joint distributions are very hard to come by and have mainly been studied in a covariance framework (e.g., Denman and Platt, 1975; Denman, 1976; Denman and Abbott, 1988, 1994), it is believed that the combination of such novel techniques and the development of integrated instrument platform for coupled biological and physical measurements (e.g., Wiebe et al., 2002; Wolk et al., 2002) would provide new insights into the nature of biophysical couplings. The journey of intermittency to elucidate oceanic processes complexity is still in its infancy.

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