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Scaling analysis of vectorial intermittency in geophysical turbulence

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1 Introduction: intermittency of vectorial time series

In fully developed turbulence, velocity and passive scalar intermittencies are often characterized through their structure functions scaling exponents $\zeta(q)$ where $< \Delta V_\ell^q > \approx \ell^{\zeta(q)}$, and $\Delta V_\ell = |V(x+\ell) - V(x)|$ is the amplitude of the increments of the field V. This framework has been shown to apply to many different velocity and passive scalar turbulent time series (invoking Taylor's hypothesis to transform fixed point time series into spatial information). Several cascade models have been proposed, leading to different types of analytical expressions for the second characteristic function $\zeta(q)$; among these, continuous cascade models, corresponding to log-infinitely divisible (log-ID) models, are now believed to be the more realistic since they involve no characteristic scale ratio. These include log-stable, log-normal, log-Poisson models. These models are able to fit experimental data up to moments of order 7 or 8 [1] (see also [2]). This is to be compared to the quite popular deterministic modeling of turbulence using eddy-viscosity models (such as $K-\epsilon$, see [3] for a review), where predictions are of moderate quality for the moment of order 1, and are often quite bad for moments of order 2 and higher. On the other hand, the latter framework is fully tensorial and can take into account boundary effects, while the intermittency/cascade multifractal stochastic framework is still limited to the analysis of 1D times series. It is then clear that a generalization of this framework to vectorial and tensorial fields is necessary. As a first step in this direction, we undertake here such extension of the 1D classical analysis, with a scaling analysis of vectorial time series. This is often available in geophysics through e.g. Doppler velocimeters giving the three components of the velocity vector at a relatively high sampling rate (10 to 100 Hz), averaged over a small volume. We first propose here a theoretical framework to deal with vectorial time series of the form $\vec{V}(t)$ at a fixed location, where $\vec{V} = (u, v)$ is the horizontal velocity vector.

In a scaling framework, each component possesses scaling properties, together with mixed moments of the form:

$$< \Delta u_\ell^p \Delta v_\ell^q > \approx \ell^{z(p,q)} \qquad (1)$$

where $p > 0$, $q > 0$, $z(p,0) = \zeta_u(p)$ and $z(0,q) = \zeta_v(q)$. This "generalized correlation function" (already studied in [4]) may be normalized by the usual structure functions in order to directly detect the degree of dependence of the two processes. This leads to study the following scaling exponents:

$$\frac{< \Delta u_\ell^p \Delta v_\ell^q >}{< \Delta u_\ell^p >< \Delta v_\ell^q >} \approx \ell^{-r(p,q)} \qquad (2)$$

For the lognormal case, $r(p,q)$ takes a simple form:

$$r(p,q) = \sigma_{uv} pq \qquad (3)$$

see textbooks on multivariate stochastic processes, such as e.g. [5]. For the general log-ID case, the expression is less simple, but it is natual to assume here, for simplicity, a polar separation of variables, with the introduction of the amplitude and direction of the moment-vector (p,q):

$$r(\rho,\phi) = f(\rho)G(\phi) \qquad (4)$$

where in the polar representation for statistical moments, we have $p = \rho\cos\phi$ and $q = \rho\sin\phi$. With this notation, for a lognormal vectorial process, $f(\rho) = \sigma\rho^2$ and $G(\phi) = \sin 2\phi$. The functions f and G may be experimentally estimated, first by taking ρ fixed and varying ϕ, and displaying the resulting curves. This is applied below to experimental geophysical turbulence.

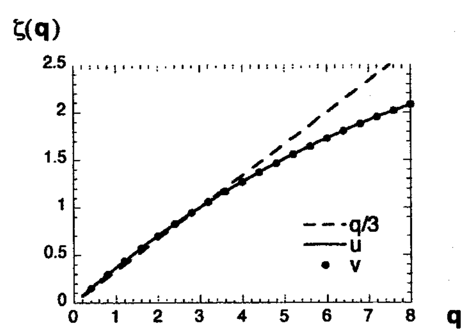

Figure 1: The scaling cumulant generating function $\zeta(q)$ for the u and v components of the vectorial data: there is a perfect superposition.

2 Application to geophysical turbulence

We apply the previous analysis approach to atmospheric and oceanic turbulent times series. Atmospheric data have been recorded at $10\,Hz$, $25\,m$ from the ground using a sonic anemometer, providing vectorial velocity times series. The

same analysis has been performed on oceanographic series recorded at $100\,Hz$ using a 3D Sontek ponctual Doppler velocimeter; only atmospheric results are displayed here. Figure 1 shows first that the scaling exponents $\zeta(q)$ of the u and v components are identical for the scaling range considered (about 3 decades). We then apply the previous procedure on the horizontal velocity vector. We show in Fig. 2 the mixed scaling exponent $r(\rho,\phi)$ vs. ϕ for several values of ρ estimated for atmospheric velocity vectorial time series. The curves are symmetric, showing the statistical isotropy of $u-v$ components. In Fig. 3 these curves are normalized by their maximum; the curves superpose perfectly, confirming the hypothesis of separation of variables (Eq. (4)). In the same Figure, the lognormal curve is shown for comparison; the fit is rather good. Furthermore, the rescaling factor has been compared to the lornormal expression $f(\rho) = \sigma\rho^2$ with a quite good fit (not shown here). This indicates that to a first approximation, the scaling of the correlations is well represented by a bivariate lognormal process. This does not necessarily mean that the process itself is a multiaffine lognormal process, since normalizations were performed.

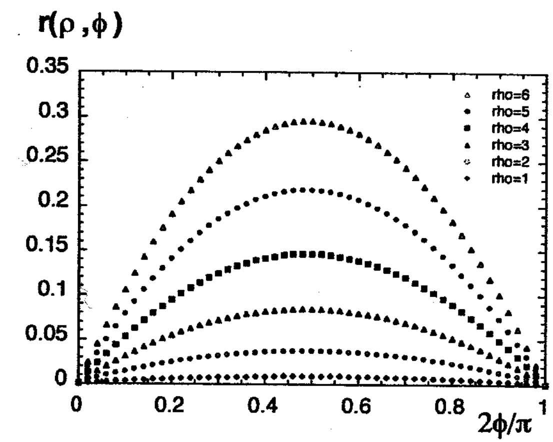

Figure 2: The mixed scaling exponent $r(\rho,\phi)$ vs. ϕ for several values of ρ estimated for atmospheric velocity vectorial time series.

3 Comments and conclusion

Our results indicate the general applicability of the scaling bivariate approach, with a separation of variables in polar coordinates. This shows also the usefulness of such an approach, since strong correlations exist between the two components of the vector: in case of independence of the components, the residual function r should be flat. We stress finally that, among many applications, this provides